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Note

The intricacy of avoiding arrays is 2

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Abstract

Let A be any $n \times n$ array on the symbols $[n]$, with at most one symbol in each cell. An $n \times n$ Latin square L *avoids* A if all entries in L differ from the corresponding entries in A. If A is split into two arrays B and C in a special way, there are Latin squares L_B and L_C avoiding B and C, respectively. In other words, the *intricacy* of avoiding arrays is 2, the number of arrays into which A has to be split.

Keywords: Latin square; Intricacy; Array

1. The theorem

The concept of *intricacy* (for completing partial Latin squares) was introduced by Daykin and Häggkvist in [2], and a sample of applications to other problems can be found in [3]. An array A is *avoidable* iff there is a Latin square L that differs from A in every cell. For the problem at hand, the intricacy is the natural number that answers the following question: "If we want to split an array into avoidable arrays, what is the maximum number of arrays we need to use?" In [1] it is proven that this number is at most 3.

There are unavoidable arrays, for example any array containing a whole row or column of just one symbol, so the intricacy is not 1.

Theorem 1. *The intricacy of avoiding arrays is* 2.

Proof. Let A be any $n \times n$ array on the symbols $[n]$. Split A into arrays B and C, so that C is empty. Certainly, there is a Latin square L_C avoiding C. For each cell in B, move the entry to array C iff it differs from the corresponding entry in L_C. Then L_C will still avoid C, and the entries left in B form a partial Latin square, which is completable (to L_C, for instance). By Theorem 2.1 in [1] B is avoidable, and is avoided by some Latin square L_B, which in fact is L_C with symbols permuted without fixed points.

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